**APPENDICES**

## Appendix A - Binomial Kernel and Difference of Gaussians Operator

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|---|
| 1 |

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 4 | 2 |
| 1 | 2 | 1 |

| | | | |
|---|---|---|---|
| 1 | 3 | 3 | 1 |
| 3 | 9 | 9 | 3 |
| 3 | 9 | 9 | 3 |
| 1 | 3 | 3 | 1 |

| | | | | |
|---|---|---|---|---|
| 1 | 4 | 6 | 4 | 1 |
| 4 | 16 | 24 | 16 | 4 |
| 6 | 24 | 36 | 24 | 6 |
| 4 | 16 | 24 | 16 | 4 |
| 1 | 4 | 6 | 4 | 1 |

**Fig. A.1** Results of applying binomial kernel in 2D, shown with dark outline after an even number of applications.

The binomial kernel is a 4x4 box filled with 1's in 2D (or an 8x8 cube in 3D). When applied repeatedly the results approximate the application of a Gaussian kernel by the Central Limit Theorem (see Fig. A.1). To keep the results centered, the kernel is applied twice, alternating the direction of displacement. Numbers shown in Fig. A.1 do not reflect normalization, which is accomplished by dividing by 4 after each iteration in 2D (or by 8 in 3D). The number of applications determines the width of the Gaussian.

The Difference of Gaussian (DOG) is achieved by subtracting the binomial kernel from itself at a distance along one of the cardinal axes proportional to the number of applications. The size of the underlying Gaussian convolution kernel, and the proportional distance at which the results are subtracted, determine the scale of the resulting partial derivative approximation. A gradient in *m* dimensions is achieved by applying a series of *m* DOG kernels, one along each cardinal axis.

| | | |
|---|---|---|
| -1 | 0 | 1 |

| | | | | |
|---|---|---|---|---|
| -1 | -2 | 0 | 2 | 1 |
| -2 | -4 | 0 | 4 | 2 |
| -1 | -2 | 0 | 2 | 1 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| -1 | -4 | -6 | -4 | 0 | 4 | 6 | 4 | 1 |
| -4 | -16 | -24 | -16 | 0 | 16 | 24 | 16 | 4 |
| -6 | -24 | -36 | -24 | 0 | 24 | 36 | 24 | 6 |
| -4 | -16 | -24 | -16 | 0 | 16 | 24 | 16 | 4 |
| -1 | -4 | -6 | -4 | 0 | 4 | 6 | 4 | 1 |

**Fig. A.2** Difference of Gaussian (DOG) for binomial kernels (dark outline in Fig A.1).

# REFERENCES


Al-Khawaja, I. M., A. Lahiri, et al. (1988). “Measurement of absolute left ventricular volume by radionuclide angiography: A technical review.” Nuclear Medicine Communications **9**: 495-504.

Amenta, N., M. Bern, et al. (1998). A new Voronoi-based surface reconstruction algorithm. Computer Graphics Proceedings, SIGGRAPH, Orlando, Florida,

Marr, D. (1982). Vision. W. H. Freeman & Co.

Bardinet, E., L. D. Cohen, et al. (1996). “Tracking and motion analysis of the left ventricle with deformable superquadrics.” Medical Image Analysis **1**(2): 129-149.

Blum, H. and R. N. Nagel (1978). “Shape description using weighted symmetric axis features.” Pattern Recognition **10**: 167-180.

Bohs, L., B. Friemel, et al. (1993). “A real time system for quantifying and displaying two-dimensional velocities using ultrasound.” Ultrasound in Medicine and Biology **19**(9): 751-761.

Brady, M. (1983). Criteria for representations of shape. New York, Academic Press.

Burbeck, C. A. and S. M. Pizer (1995). “Object representation by cores: Identifying and representing primitive spatial regions.” Vision Research **35**(13): 1917-1930.

Chalana, V., D. T. Linker, et al. (1996). “A multiple active contour model for cardiac boundary detection on echocardiographic sequence.” IEEE Transactions on Medical Image **15**(3): 290-298.

Clary, G. J., S. M. Pizer, et al. (1997). Left ventricular wall motion tracking via deformable shape loci. CAR International Symposium and Exhibition, Berlin, Elsevier Science B.V.

Clarysse, P., D. Friboulet, et al. (1997). “Tracking geometrical descriptors on 3-D deformable surfaces: Application to the left-ventricular surface of the heart..” IEEE Transactions on Medical Imaging **16**(4): 392-404.

Cootes, T. F., C. J. Taylor, et al. (1994). “Active Shape Models - Their Training and Application.” Computer Vision and Image Understanding **61**(1): 38-57.

Coppini, G., R. Poli, et al. (1995). “Recovery of the 3-D shape of the left ventricle from echocardiographic images.” IEEE Transactions on Medical Imaging **14**(2): 301-317.

Culver, T. (1998). Accurate computation of the medial axis of a polyhedron. TR98-034, Department of Computer Science, UNC, Chapel Hill. (submitted to ACM Solid Modeling).

Debatin, J. F., S. N. Nadel, et al. (1992). “Cardiac ejection fraction: Phantom study comparing cine MR imaging, radionuclide blood pool imaging, and ventriculography.” Journal of Magnetic Resonance Imaging **2**: 135-142.

Dodge, H. T., H. Sandler, et al. (1960). "The use of biplane angiography for measurement of left ventricular volume in man." American Heart Journal **60**: 762-776.

Eberly, D. and J. Lancaster (1991, November). "On gray scale image measurements: I. Arc length and area." CVGIP: Graphical Models and Image Processing **53**(6): 538-549.

Eberly, D., J. Lancaster, et al. (1991, November). "On gray scale image measurements: II. Surface area and volume." CVGIP: Graphical models and image processing **53**(6): 550-562.

Fazzalari, N., J. Davidson, et al. (1984). "Three dimensional reconstruction of the left ventricle from four anatomically defined apical two-dimensional echocardiographic views." Acta Cardiologica **39**(6): 409-36.

Feng, J., W. C. Lin, et al. (1991). "Epicardial Boundary Detection Using Fuzzy Reasoning." IEEE Transactions on Medical Imaging **10**(2): 187-199.

Fleishman, C. E., J. Li, et al. (1996). Identification of congenital heart defects using real-time three-dimensional echo in pediatric patients. American Heart Association 69th Scientific Sessions.

Fleishman, C. E., T. Ota, et al. (1996). Real-time three-dimensional echocardiography: Measurement of left ventricular mass in dogs. American Heart Association 69th Scientific Sessions.

Fritsch, D., S. Pizer, et al. (1997). Segmentation of Medical Image Objects Using Deformable Shape Loci. Information Processing in Medical Imaging, Poultney, Vermont, Springer.

Furst, J. D. and S. M. Pizer (1998). Marching optimal-parameter ridges: An algorithm to extract shape loci in 3D images. Medical Image Computing and Computer-Assisted Intervention, Cambridge, MA, Springer.

Furst, J. D. and S. M. Pizer (June 1996). Marching Cores: A method for extracting cores from 3D medical images. Proceedings of the Workshop on Mathematical Methods in Biomedical Image Analysis, San Francisco, CA.

Geiser, E. (1985). "Three-dimensional echocardiographic reconstruction: how does it stack up?" International Journal of Cardiology **7**: 77-81.

Gopal, A., D. King, et al. (1992). "Three-dimensional echocardiographic volume computation by polyhedral surface reconstruction: in vitro validation and comparison to magnetic resonance imaging." Journal of the American Society of Echocardiography **5**(2): 115-124.

Hozumi, T., K. Yoshida, et al. (1997). "Echocardiographic estimation of left ventricular cavity area with a newly developed automated contour tracking method." Journal of the American Society of Echocardiography **10**(8): 822-829.

Hsieh, A., M. Collins, et al. (1996, (submitted)). Real-time 3D stress echocardiography: Description of a new technique. American Heart Association 69th Scientific Sessions,

Hunter, I. A., J. J. Soraghan, et al. (1993). Detection of Echocardiographic Left Ventricle Boundaries Using Neural Networks. Computers in Cardiology, London, UK,

J. Baan, E.T. van der Velde, et al. (1984). “Continuous measurement of left ventricular volume in animals and humans by conductance catheter.” Circulation **70**(5): 812-823.

Kaplan, W. (1973). Advanced Calculus. Reading, Mass., Addison-Wesley.

Koenderink, J. J. (1990). Solid Shape. Cambridge, MA, MIT Press.

Leyton, M. (1992). Symmetry, Causality, and Mind. Cambridge, MA, MIT Press.

Low, K-C. and J. Coggins, "Biomedical Image Segmentation Using Multiscale Orientation Fields," First International Conference on Visualization in Biomedical Computing, Atlanta, May 22-25, 1990, pp. 378-384.

Malassiotis, S. and M. G. Strintzis (1999). “Tracking the left ventricle in echocardiographic images by learning heart dynamics.” IEEE Transactions on Medical Imaging **18**(3): 282-290.

Maddahi, J. (1998). "Nuclear cardiology: Which clinical questions can be answered now and in the future?" What's new in cardiovascular imaging?, edited by Reiber and van der Wall, Dordrecht, Kluwer, Chapter 20, 261-291.

Matsumura, K., E. Nakase, et al. (1993). “Determination of cardiac ejection fraction and left ventricular volume: contrast-enhanced ultrafast cine MR imaging vs IV digital subtraction ventriculography.” American Journal of Roentgenology **160**(5): 979-85.

McInerney, T. and D. Terzopoulos (1996). “Deformable models in medical image analysis: a survey.” Medical Image Analysis **1**(2): 91-108.

Mikic, I., S. Krucinski, et al. (1998). “Segmentation and tracking in echocardiographic sequences: Active contours guided by optical flow estimates.” IEEE Transactions on Medical Imaging **17**(2): 274-284.

Moritz, W., A. Pearlman, et al. (1982). “An ultrasonic technique for imaging the ventricle in three dimensions and calculating its volume.” IEEE Transactions on Biomedical Engineering **30**: 482-91.

Morse, B. S., S. M. Pizer, et al. (1998). “Zoom-Invariant vision of figural shape: Effect on cores of image disturbances.” Computer Vision and Image Understanding **69**: 72-86.

Nackman, L. R. (1982). “Curvature relations in 3D symmetric axes.” CVGIP **20**: 43-57.

Nackman, L. R. and S. M. Pizer (1985). “Three-Dimensional shape description using the symmetric axis transform I:Theory.” IEEE Transactions on Pattern Analysis and Machine Intelligence **2**(2): 187-202.

O'Rourke, J. (1998). Computational Geometry in C. Cambridge, Cambridge University Press.

Ohazama, C. (1998). Ph.D. Dissertation Research, Duke University.

Ohazama, C. J., G. Stetten, et al. (1996, (in press)). A new, rapid visualization method for detection of ischemic risk volume by three-dimensional echocardiography. American Heart Association 69th Scientific Sessions, Ota, T., C. E. Fleishman, et al. (1996). Accuracy of manual and semiautomated methods for volume determination by real-time, three-dimensional echocardiography. American Heart Association 69th Scientific Sessions,

Ota, T., C. E. Fleishman, et al. (1996). Measurement of left ventricular volume by real-time, three-dimensional echocardiography in dogs. American Heart Association 69th Scientific Sessions,

Ota, T., C. E. Fleishman, et al. (1997). Manual and semiautomated measurement of left ventricular volume using real-time, three-dimensional echocardiography in vivo. *American College of Cardiology Annual 46th Scientific Session*,

Ota, T., C. J. Ohazama, et al. (1997). Novel determination of left ventricular volume by tracing arbitrary planes using real-time, 3D echocardiography: in vitro and in vivo validation. AHA 70th Scientific Sessions,

Park, J., D. Metaxas, et al. (1996). "Deformable Models with Parameter Functions for Cardiac Motion Analysis from Tagged MRI Data." IEEE Transactions on Medical Imaging **15**(3): 278-289.

Pizer, S. M., D. H. Eberly, et al. (1998). "Zoom invariant vision of figural shape: the mathematics of cores." Computer Vision and Image Understanding **69**(1): 55-71.

Pizer, S. M., D. S. Fritsch, et al. (1999). "Segmentation, Registration, and Measurement of Shape Variation via Image Object Shape." IEEE Transactions on Medical Imaging **18**(10).

Pizer, S., A. Thall, et al. (submitted 1999). "M-Reps: A new object object representation for graphics." ACM Transactions on Graphics :

Raichlen, J. S., S. S. Trivedi, et al. (1986). "Dynamic three-dimensional reconstruction of the left ventricle from two-dimensional echocardiograms." Journal of the American College of Cardiology **8**: 364-370.

Robb, R. A. (1983). "High-speed three-dimensional x-ray computed tomography: The Dyamic Spatial Reconstructor." Proceedings of the IEEE **71**: 308-319.

Rumberger, J. A. and M. R. Bell (1992). "Measurement of Myocardial Perfusion and Cardiac Output Using Intravenous Injection Methods by Ultrafast (Cine) Computed Tomography." Investigative Radiology **27**: S40-S45.

Shattuck, D. P., M. D. Weinshenker, et al. (1984). "Explososcan: A parallel processing technique for high speed ultrasound imaging with linear phased arrays." Journal of the Acoustical Society of America **75**(4): 1273-1282.

Sheikh, K., S. Smith, et al. (1991). "Real-Time, Three Dimensional Echocardiography: Feasibility and Initial Use." Echocardiography **8**(1): 119-125.

Smith, S. W., H. G. Pavy, et al. (1992). "Improved real-time volumetric ultrasonic imaging." Ultrasonic Imaging **14**: 186-211.

Smith, S. W., J. Henry G. Pavy, et al. (1991). "High-speed ultrasound volumetric imaging system - Part I: Transducer design and beam steering." IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control **38**(2): 100-108.

Stadius, M. L. and J. L. Richie (1986). Gated Blood Pool Tomography. New Concepts in Cardiac Imaging. Chicago, Year Book Medical Publishers, Inc. 137-154.

Staib, L. H. and J. S. Duncan (1996). “Model-based deformable surface finding for medical images.” IEEE Transactions on Medical Imaging **15**(5): 1-12.

Stetten, G. and R. Morris (1995). “Shape detection with the Flow Integration Transform.” Information Sciences **85**: 203-221.

Stetten, G. and S. Pizer (1998). "Extracting Shape Properties via Populations of Medial Primitives." University of North Carolina, Department of Computer Science, TR98-008.

Stetten, G. and S. Pizer (1999). "Automated Identification and Measurement of Objects via Populations of Medial Primitives, with Application to Real Time 3D Echocardiography." XVIth International Conference on Information Processing in Medical Imaging, June, 1999. Lecture Notes in Computer Science, vol. 1613, pp. 84-97)

Stetten, G. D. and S. M. Pizer (1999). “Medial Node Models to Identify and Measure Objects in Real-Time 3D Echocardiography.” IEEE Transactions on Medical Imaging **18**(10): 1025-1034**.**

Stetten, G., M. Caines, et al. (1995). “The Volumetricardiogram (VCG): Volume Determination of Cardiac Chambers using 3D Matrix-Array Ultrasound.” Proceedings of the SPIE Symposium on Medical Imaging **2432**: 185-196.

Stetten, G., R. Landesman, et al. (1997). “Core-Atoms and the spectra of scale.” SPIE Medical Imaging Conference **3034, part 2**: 642-652.

Stetten, G., T. Irvine, et al. (1999 (in press)). Improved accuracy for a semi-automated method for computing right ventricle (RV) cavity volumes from Real Time 3D Echo: Comparison studies to ultrasonic crystals in an open-chest animal model. *American College of Cardiology 48th Scientific Sessions*, New Orleans,

Stetten, G., T. Ota, et al. (1998). “Real-Time 3D Ultrasound: A New Look at the Heart.” Journal of Cardiovascular Diagnosis and Procedures **15**(2): 73-84.

Szekely, G., A. Kelemen, et al. (1996). “Segmentation of 3D objects from MRI volume data using contrained elastic deformations of flexible Fourier surface models.” Medical Image Analysis **1**(1): 19-34.

Trahey, G., J. Allison, et al. (1987). “Angle independent ultrasonic detection of blood flow.” IEEE Transactions on Biomedical Engineering **34**: 965-967.

von Ramm, O. T. (1973). A real time digitally controlled ultrasound imaging system. Ph.D. Dissertation, Duke University Department of Biomedical Engineering.

von Ramm, O. T. and F. L. Thurstone (1976). “Cardiac imaging using a phased array system (I. System design).” Circulation **53**(2): 258-262.

von Ramm, O. T., S. W. Smith, et al. (1991). “High-speed ultrasound volumetric imaging system - Part II: Parallel processing and image display.” IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control **38**(2): 109-115.

von Ramm, O. T. v., H. A. Pavy, et al. (1991). “Real-Time three dimensional echocardiography: The first human images.” Circulation abstract

Wilson, D. C., E. A. Geiser, et al. (1993). “Feature extraction in two-dimensional short-axis echocardiographic images.” Journal of Mathematical Imaging and Vision **3**: 285-298.

Wynne, J., L. Green, et al. (1978). “Estimation of left ventricular volumes in man from biplane cineangiograms filmed in oblique projections.” American Journal of Cardiology **41**: 726-32.

Zoghbi, W. A., J. C. Buckey, et al. (1990). “Determination of left ventricular volumes with use of a new nongeometric echocardiographic method: clinical validation and potential application.” Journal of the American College of Cardiology **15**(3): 610-617.