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Discrete Period Quadrature for Time-Frequency Analysis of Nystagmus Eye Motion

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Introduction: Nystagmus, a condition characterized by involuntary, repetitive eye motion, impacts about 785,000 individuals in the United States alone and can cause oscillopsia, or the perception of a moving visual field. Treatment options, including corrective eyeglasses or contact lenses, medications, surgery, and rehabilitation therapy, often have inconsistent success or are prohibitively expensive. As a possible solution, we propose a mobile application (app), StabilEyes, for smart devices. It will use the front-facing camera of the smart phone or tablet to detect the user's periodic eye motion and stabilize real-time images from the back-facing camera by translating those images on the device's screen. Previously, we reported an effective method to detect periodic eye motion using first-order moment calculations on images of the face [1]. We have now developed a method, Discrete Period Quadrature (DPQ), to efficiently and accurately track the frequency and phase of that periodic signal.

Materials and Methods: Similar to the Discrete Fourier Transform (DFT), DPQ operates on a discrete signal, $s[n]$, yielding complex values for a set of frequencies. However, unlike the DFT, DPQ utilizes a window whose size adapts to exactly one discrete period for each frequency considered. This optimizes DPQ for time-frequency analysis, in our case, rapid and accurate tracking of a periodic signal. We denote DPQ as $Q[p,n]$ and define it as

$$Q[p,n] = \sum_{m=n}^{n-p+1} \frac{s[m]\left(\cos\left(\frac{2\pi(m-n)}{p}\right)+j\sin\left(\frac{2\pi(m-n)}{p}\right)\right)}{p}, \quad 2 \le p \le P,$$

where the sum operates on p samples preceding and including the sample at index n. For each integer period p between 2 and a maximum P, this yields the quadrature covariance at each discrete frequency $\omega = 2\pi/p$ as a complex number. The magnitude $|Q[p,n]|$ and phase $\angle Q[p,n]$ play similar roles to the magnitude and phase of the DFT but operate on a single period at each discrete frequency. Given a sinusoidal signal with a period p_s, we expect $|Q[p,n]|$ to have a maximum at $p \cong p_s$ and a minimum at $p \cong 2p_s$. We compared the capability of each of these predicted behaviors to infer the period, and thereby determine the phase, of a sinusoidal signal.

Results and Discussion: Fig. 1 shows the $|Q[p,n]|$ spectrum at some index n_i. Truncation errors occurring when correlating with periods near p_s cause the location of the spectrum's peak to fluctuate. In contrast, the magnitude of $2p_s$ is consistently at a trough, near zero. Fig. 2 compares the accuracy of using the peak and the 2nd subharmonic ($2p_s$) to track a signal whose period undergoes a change, as a function of the rate of change.

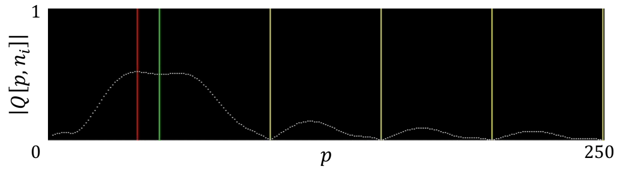

Figure 1. Magnitude spectrum at $p_s = 50$. The green, red, and yellow lines denote p_s, the value of p with maximum $|Q[p,n_i]|$, and 2nd - 5th subharmonics of p_s, respectively.

Conclusions: As shown in Fig. 2, inferring p_s using the minimum $|Q[p,n]|$ at $2p_s$ yields more accurate tracking than using the maximum $|Q[p,n]|$ at p_s. Given an input signal derived from our previously developed eye-tracking method [1], we will employ DPQ to determine the fundamental period and use the corresponding phase to control the image translation in our StabilEyes app. Further refinement and validation of the system should lead to eventual testing in patients with nystagmus.

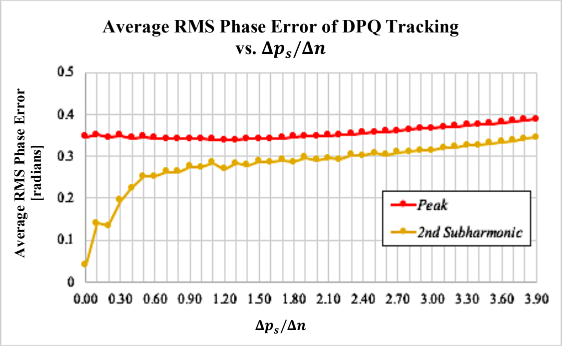

Figure 2. RMS phase error (averaged over $2 \le p_s \le 64$) as a function of the rate at which signal period p_s was changed over 5 consecutive samples.

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References:

1. J. Foust, et al., "StabilEyes - New Assistive Technology for Nystagmus to Produce a Stabile Real-Time Video Image" at BMES 2019 Annual Meeting, Philadelphia, PA, 2019.