Evaluating Discrete Period Quadrature for Time-Frequency Analysis using Various Features of the Magnitude and Phase Spectra

Visualization and Image Analysis Laboratory

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Background

- Nystagmus: visual impairment characterized by involuntary, repetitive eye motion
  - Perception of unstable visual field - *oscillopsia*
- *StabilEyes*: mobile application to compensate for unwanted nystagmus eye motion
  - "Shift" on-screen images at the correct frequency and phase in real time
- Need efficient, accurate method of tracking frequency and phase of periodic eye motion

Purpose of Study

- Evaluate the use of four spectral features produced by Discrete Period Quadrature (DPQ)
  1. *Global maximum magnitude at signal period*
  2. *Local minimum magnitudes at subharmonics of signal period*
  3. *Global minimum variance of magnitude at signal period*
  4. *Global minimum RMS delta phase error at signal period*

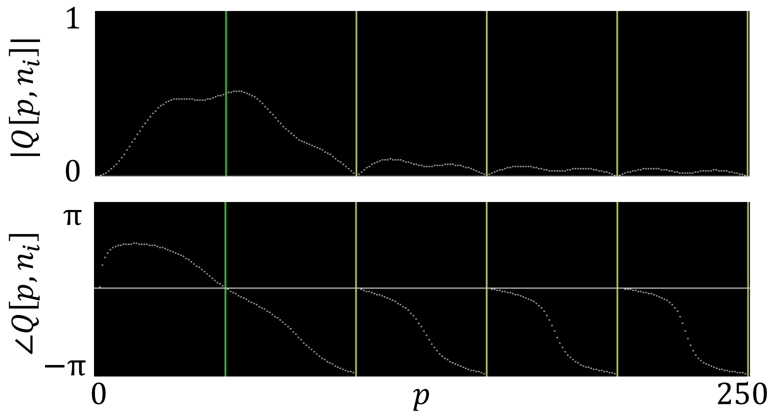
Methods

- DPQ definition, for each period p and (discrete time) index n, for discrete signal $s[n]$:

$$Q[p,n] = \sum_{m=n}^{n-p+1} \frac{s[m]\left(\cos\left(\frac{2\pi(m-n)}{p}\right) + j\sin\left(\frac{2\pi(m-n)}{p}\right)\right)}{p}, 2 \leq p \leq P$$

- Signal period p_s inferred using one of the following features:

  1. Peak:
     $|Q[p,n]|$ has global maximum at $p \cong p_s$
  2. Subharmonic:
     $|Q[p,n]|$ has local minima at $p \cong kp_s,\ k = 2, 3, 4, \ldots$
  3. Variance:
     $VAR(|Q[p,n]|)$ has local minima at $p \cong kp_s,\ \ k = 1, 2, 3, \ldots$
  4. Delta Phase:
     RMS error of $[\angle Q[p,n] - \angle Q[p,n-1]] - \frac{2\pi}{p}$ has global minimum at $p \cong p_s$

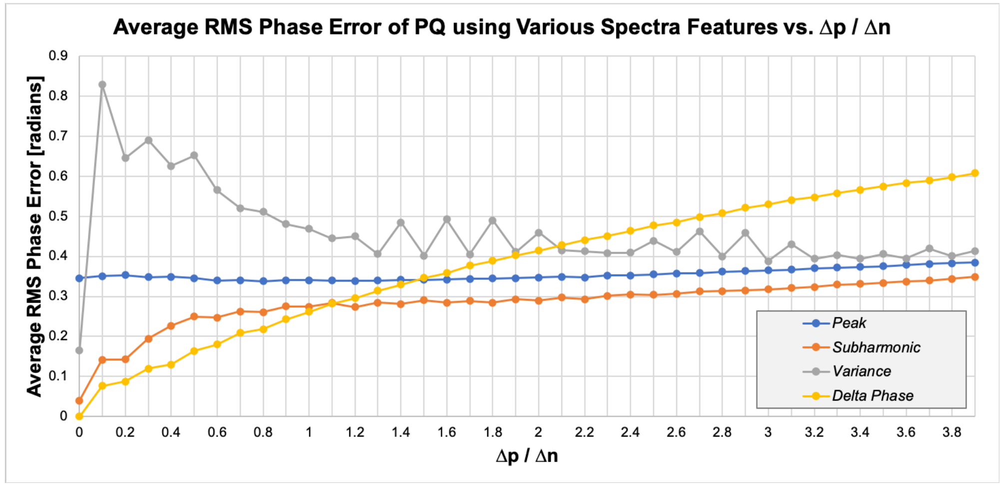


Magnitude and phase spectra at $p_s = 50$. The green and yellow lines denote p_s and its subharmonics.

- Experimental: using sinusoidal signal with **(1) changing signal period & (2) Gaussian Noise**
- Observational: real-time with interactive **non-sinusoidal waveform (sawtooth)**

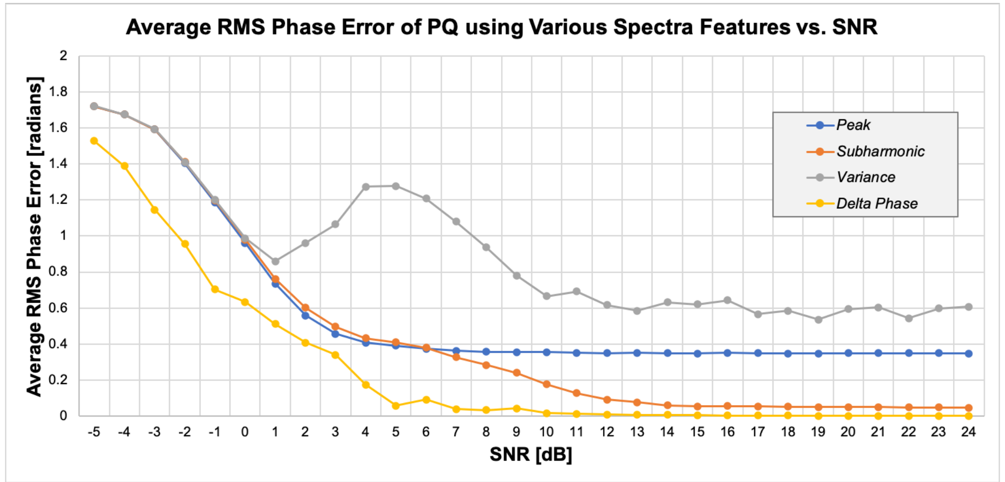
Results & Conclusions

Changing Signal Period ($\frac{\Delta p_s}{\Delta n}$)



Delta phase: smallest error between true signal and inferred phases until rate of change $\frac{\Delta p_s}{\Delta n} \approx 1$

Noisy Signal



Delta phase: smallest error between true signal and inferred phases for all SNRs tested

- Interactive testing with non-sinusoidal signals – delta phase performs best, most **consistently**
- Delta phase has some **memory** to aid in tracking, especially in the presence of noise
- Next step: analyze and characterize **DPQ using delta phase to infer period**